ATMAM Mathematics Methods

Test 1 2020 Calculator Free

SHENTON COLLEGE

Name: ......SOLUTIONS......

Teacher (Please circle name) Ai Friday White

Time Allowed : 30 minutes

Marks	/35

Materials allowed: Formula Sheet.

Attempt all questions. Questions 1,2, 3 ,4, 5 and 6 are contained in this section.
All necessary working and reasoning must be shown for full marks.
Where appropriate, answers should be given as exact values. Marks may not be awarded for untidy or poorly arranged work.

1. [2+2+2+3=9 marks]

Determine the derivative of each of the following with respect to x of, clearly showing use of appropriate rules. Do not simplify your answers.

(a) $y = (2x^3 + 1)(5x - 16)$

$$\frac{dy}{dx} = 5(2x^3+1) + (6x^2)(5x-16)$$

✓ demonstrates use of product rule

✓ correct differentiation

(b) $y = \sqrt[3]{(x^2 - 4x)}$

$$\frac{dy}{dx} = \frac{1}{3}(x^2-4x)^{-\frac{2}{3}}(2x-4)$$

✓ shows use of chain rule on $\sqrt[3]{\ }$

✓ $\frac{d}{dx}(x^2-4x)$

(c) $y = \frac{x^{\frac{1}{2}}}{x+1}$

$$\frac{dy}{dx} = \frac{(x+1)\left(\frac{1}{2}x^{-\frac{1}{2}}\right) - x^{\frac{1}{2}}(1)}{(x+1)^2}$$

✓ correct use of Quotient rule in numerator demonstrated

✓ all parts correct

(d) $y = cos^3(4x + 1)$

$$\frac{dy}{dx} = 3\cos^2(4x+1)(-\sin(4x+1))(4)$$

✓ chain on $\cos^3$

✓ $\frac{d}{dx}\cos(4x+1)$

✓ $\frac{d}{dx}(4x+1)$

2. [2+2= 4 marks]

Determine:

(a) $\lim\limits_{h \to 0} \frac{(x+h)^3 - x^3}{h}$

$= \frac{d}{dx}(x^3)$ ✓ recognizes $f(x) = x^3$

$= 3x^2$ ✓ $\frac{d}{dx}(3x^2)$

(b) $\frac{d}{dt}$ 2cos(t^o)where t^o is t degrees

$= \frac{d}{dt} 2\cos\left(\frac{\pi}{180}t\right)$ ✓ $t^\circ = \frac{\pi}{180}t$

$= -\frac{2\pi}{180}\sin\left(\frac{\pi}{180}t\right)$

$= -\frac{\pi}{90}\sin t^\circ$ ✓ $\frac{d}{dt}$ in terms of t°

3. [4 marks]

Determine the equation of the tangent to $y = x\sin x$ at the point $\left(\frac{-5\pi}{2}, \frac{5\pi}{2}\right)$.

$\frac{dy}{dx} = (1)\sin x + x(\cos x)$

$= \sin x + x\cos x.$ ✓ $\frac{dy}{dx}$

$\left.\frac{dy}{dx}\right|_{x=-\frac{5\pi}{2}} = \sin\left(-\frac{5\pi}{2}\right) + \left(-\frac{5\pi}{2}\right)\cos\left(-\frac{5\pi}{2}\right)$

$= -1 + \left(-\frac{5\pi}{2}\right)(0)$ ✓ $\frac{dy}{dx}$ at $x = \left(-\frac{5\pi}{2}\right)$

$= -1$

$\therefore$ Equation of tangent $y = -x + c$ at $\left(-\frac{5\pi}{2}, \frac{5\pi}{2}\right)$

$\frac{5\pi}{2} = \frac{5\pi}{2} + c$

$c = 0$ ✓ c

$\therefore y = -x$ ✓ equation of tangent

. [3 marks]

Use the table below to help find $\frac{d}{dx}(g(h(x))$ at $x = 0$

x	$g(x)$	$h(x)$	$g'(x)$	$h'(x)$
0	5	2	-1	1
2	11	8	7	5

$$\frac{d}{dx} g(h(x)) = g'(h(x))\, h'(x)$$

$$\left.\frac{d}{dx}\right|_{x=0} = g'(h(0))\, h'(0)$$ ✓ demonstrates understanding of chain rule

$$= g'(2)\, h'(0)$$

$$= (7)\,(1)$$ ✓ correct use of table values

$$= 7$$

✓ $\frac{d}{dx}$

5 [4 marks]

If $y = \cos(2x)$, determine a simple expression for $(\frac{dy}{dx})^2 + \frac{1}{4}(\frac{d^2y}{dx^2})^2$

$$y = \cos(2x)$$

$$\frac{dy}{dx} = -2\sin(2x)$$

$$\frac{d^2y}{dx^2} = -4\cos(2x)$$

$$\therefore \left(\frac{dy}{dx}\right)^2 + \frac{1}{4}\left(\frac{d^2y}{dx^2}\right)^2$$

$$= [-2\sin(2x)]^2 + \frac{1}{4}[-4\cos(2x)]^2$$ ✓ $\left(\frac{dy}{dx}\right)^2$

$$= 4\sin^2(2x) + \frac{1}{4}[16\cos^2(2x)]$$ ✓ $\frac{d^2y}{dx^2}$

$$= 4\sin^2(2x) + 4\cos^2(2x)$$ ✓ substitution

$$= 4(\sin^2(2x) + \cos^2(2x)]$$ ✓ evaluation

$$= 4(1)$$

$$= 4$$

6. [4+4+3=11 marks]

Consider the function $f(x) = x^3(4-x)$

(a) Use calculus to determine the location of all stationary points.

$f(x) = x^3(4-x)$

$f'(x) = 3x^2(4-x) + x^3(-1)$

$= 12x^2 - 3x^3 - x^3$

$= 12x^2 - 4x^3$

$= 4x^2(3-x)$

Stationary points when $f'(x) = 0$

ie. $x = 0$ and $x = 3$

$(0,0)$ $(3,27)$

✓ $f'(x)$

✓ $f'(x) = 0$

✓ $x = 0$, $x = 3$

✓ $(0,0)$, $(3,27)$ Here or in (b)

(b) Use the second derivative to determine the nature of these stationary points.

$f''(x) = 24x - 12x^2$ ✓ $f''(x)$

$f''(0) = 0$ ∴ Possible Horizontal point of Inflection

$f''(3) < 0$ ∴ Maximum stationary point. ✓ $f''(x)$ test shown

Check Horizontal Point of Inflection for concavity change

x	-1	0	1
$f''(x)$	<0	0	>0

✓ Test for Hor P. of I Shown

∴ $(0,0)$ Horizontal point of Inflection

$(3,27)$ Maximum stationary point.

✓ $(3,27)$ Max. T.P.

(c) Determine with justification the location of any non-stationary points of inflection.

$f''(x) = 0$

$24x - 12x^2 = 0$

$12x(2-x) = 0$

$x = 0$ $x = 2$

$x = 0$ Hor P. of I

Test $x = 2$ for concavity change

x	1	2	3
$f''(x)$	>0	0	<0

✓ $f''(x) = 0$

✓ Test concavity

Concavity change ∴ Non-stationary point of inflection at $(2,16)$ ✓ $(2,16)$

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Mathematics Methods

Test 1 2020 Calculator Assumed

Name: Solutions.

Teacher (Please circle name) Ai Friday White

Time Allowed : 20 minutes

Marks	/ 19

Materials allowed: *Classpad calculator,* Formula Sheet.

Attempt all questions. Questions 7, 8 ,9 and 10 are contained in this section.
All necessary working and reasoning must be shown for full marks.
Marks may not be awarded for untidy or poorly arranged work.

7. [4 marks]

Show the use of differentiation to determine the approximate change in y when x changes from 2 to 2.1 if $y = 2\sin x + \cos x$.

$y = 2\sin x + \cos x$ $\qquad \delta x = 0.1$ ✓ δx

$\frac{dy}{dx} = 2\cos x - \sin x$

$\delta y \approx \frac{dy}{dx} \cdot \delta x$ ✓ Shows use of incremental formula

$\approx \left.\frac{dy}{dx}\right|_{x=2} 0.1$

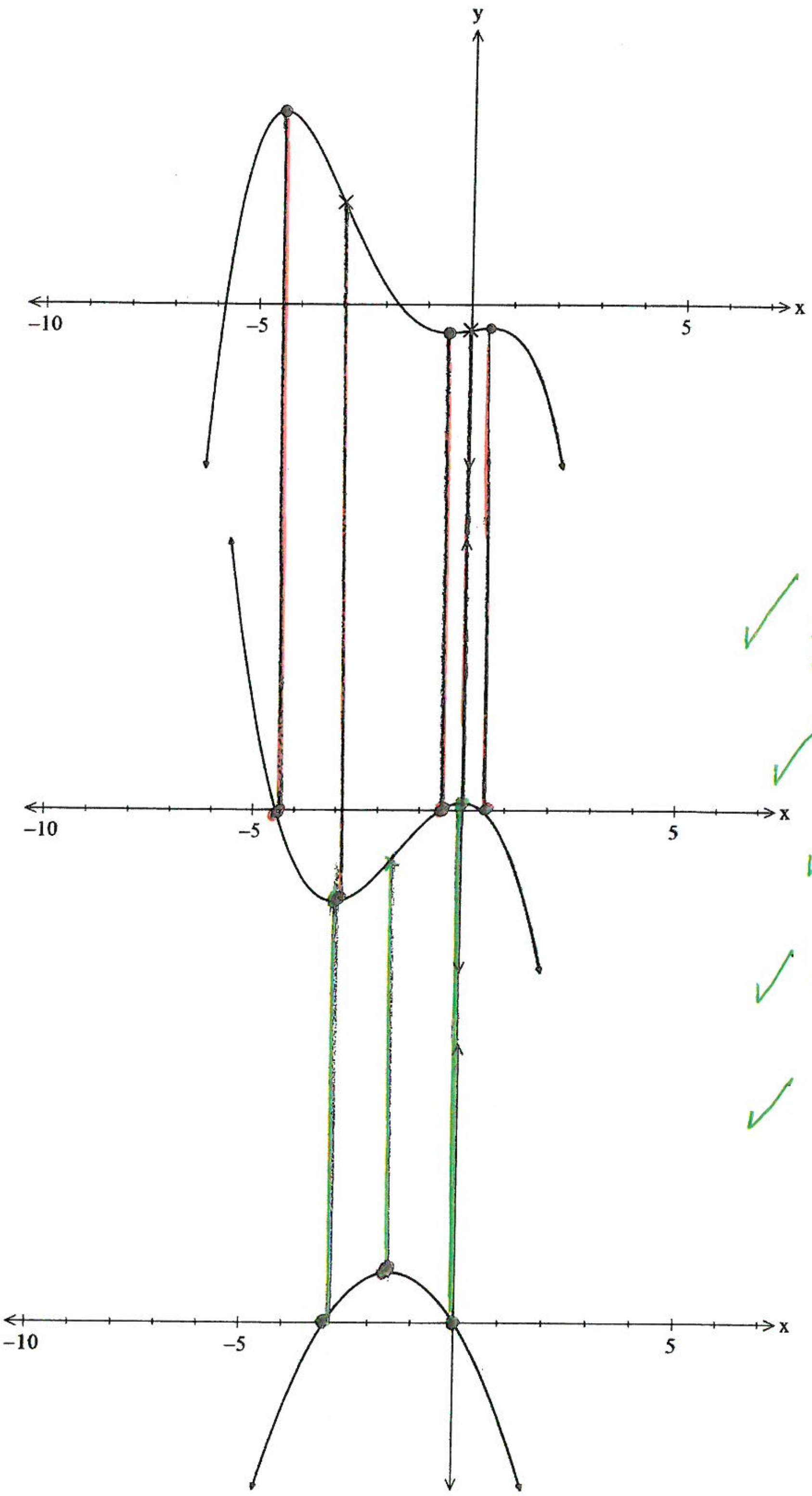
$\approx -1.7416(0.1)$

≈ -0.1742 ✓ δy.

y decreases approximately by 0.1742 when x changes from 2 to 2.1 ✓ descriptive answer

8. [3+2=5 marks]

The graph below shows a polynomial function with non-horizontal points of inflection at $x = -3 \; and \; x = 0$. On the sets of axes provided graph the first derivative and the second derivative graphs for this function, clearly indicating the relationship between relevant points.



$y = f'(x)$

✓ $\frac{dy}{dx} = 0 \rightarrow$ roots

Points of Inflection

✓ to Min

✓ to Max

$y = f''(x)$

✓ $\frac{d^2y}{dx^2} = 0 \rightarrow$ roots

✓ inflection to Max

9. [3+2+2= 7 marks]

UNITS

A particle, P, moves along the x-axis with position given by $x(t) = 5.2\sin\left(\frac{t}{2}\right) + 3$ cm where t is the time in seconds, $0 \leq t \leq 18$

(a) Determine the initial position, velocity and acceleration of P and give these values.

$x(0) = 3$ cm ✓ initial position

$v(0) = 2.6 \text{ cm s}^{-1}$ ✓ initial velocity

$a(0) = 0 \text{ cm s}^{-2}$ ✓ initial acceleration

(b) Describe the motion of the particle when $t = 3.2$ seconds

Particle moving to the left. $v(3.2) < 0$ at an increasing velocity. $a(3.2) < 0$, $v(3.2) < 0$

✓ moving left

✓ increasing velocity

$v(3.2) = -0.8 \text{ cm s}^{-1}$

$a(3.2) = -1.3 \text{ cm s}^{-2}$

(c) Determine the time or times when the particle's velocity is increasing at its fastest rate $0 \leq t \leq 18$ and explain your answer.

$t = 9.4248$ s correct 't'! ..

$v''(t) = 0$ ie. point of inflection, rate of change changing.

$v'(t) > 0$ ∴ inflection point is point on graph where rate of change is most positive (fastest)

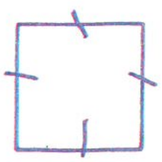
or maximum acceleration

✓ suitable reason

10. [3 marks]

A metal plate of square shape, length x cm, is heated so that its sides expand at a rate of 0.01cm/min.

$\frac{dA}{dx}$ is rate of change of the area of the square with respect to its side lengh and $\frac{dx}{dt} = 0.01$ is the rate the sides expand with respect to time. By first stating $\frac{dA}{dx}$, show how to use the chain rule to obtain $\frac{dA}{dt}$, the rate of change of the plate's area with respect to time . Evaluate this rate when the side of the square is 10 cm.



x cm

$A = x^2$

$\frac{dA}{dx} = 2x$

$\frac{dA}{dt} = \frac{dA}{dx} \cdot \frac{dx}{dt}$ ✓ show use of chain rule

$= 2x \cdot (0.01)$ ✓ correct rates

$\left.\frac{dA}{dt}\right|_{x=10} = 0.2 \text{ cm}^2/\text{min}$ ✓ evaluate rate when $x = 10$